

Quark mass dependence of nucleon observables and lattice QCD

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- $SU(2)$ Baryon Chiral Perturbation Theory with and without explicit Δ (1232)
- Quark mass dependence of M_N and g_A to one-loop order
- Numerical analysis using lattice QCD data

T.R. Hemmert, M. P. and W. Weise, PRD 68 (2003) 075009

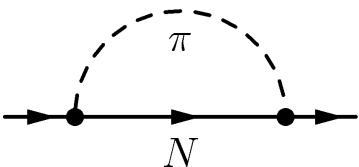
M. P., T.R. Hemmert and W. Weise, PRD 69 (2004) 034505

M. P., B.U. Musch, T. Wollenweber, T.R. Hemmert and W. Weise, *forthcoming*.

M_N leading one-loop $\mathcal{O}(p^3)$: analytic results πN

quark → pion mass dependence according to GOR relation

Infrared Regularization (BL 99)



and $\mathcal{L}_{\pi N \text{ c.t.}}^{(4)} = -4 e_1 m_\pi^4 \bar{\Psi} \Psi$

$$M_N = M_0 - 4 c_1 m_\pi^2 + \left[4 e_1^r(\lambda) + \frac{3 g_A^0{}^2}{64 \pi^2 f_\pi^0{}^2 M_0} \left(1 - 2 \ln \frac{m_\pi}{\lambda} \right) \right] m_\pi^4$$

$$- \frac{3 g_A^0{}^2}{16 \pi^2 f_\pi^0{}^2} m_\pi^3 \sqrt{1 - \frac{m_\pi^2}{4 M_0^2}} \arccos \left(-\frac{m_\pi}{2 M_0} \right)$$

and expanding around the chiral limit:

$$M_N = M_0 - 4 c_1 m_\pi^2 - \frac{3 g_A^0{}^2}{32 \pi f_\pi^0{}^2} m_\pi^3 + \left[4 e_1^r(\lambda) - \frac{3 g_A^0{}^2}{64 \pi^2 f_\pi^0{}^2 M_0} \left(1 + 2 \ln \frac{m_\pi}{\lambda} \right) \right] m_\pi^4$$

$$+ \frac{3 g_A^0{}^2}{256 \pi f_\pi^0{}^2 M_0^2} m_\pi^5 + \mathcal{O}(m_\pi^6)$$

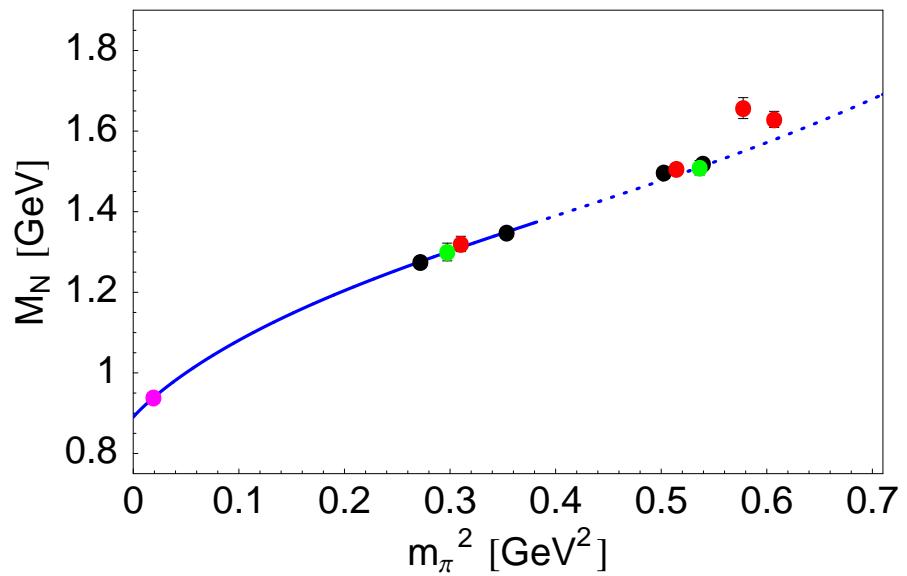
In the numerical evaluation: $g_A^0 \rightarrow g_A^{\text{phys}} = 1.267$, $f_\pi^0 \rightarrow f_\pi^{\text{phys}} = 92.4 \text{ MeV}$, $\lambda = 1 \text{ GeV}$

M_N to leading one-loop: numerical results πN

$N_f = 2$ full-QCD lattice data

CP-PACS, JLQCD and UKQCD-QCDSF (2002 & 2003) $a < 0.15 \text{ fm}$, $m_\pi L > 5$, $m_\pi < 600 \text{ MeV}$

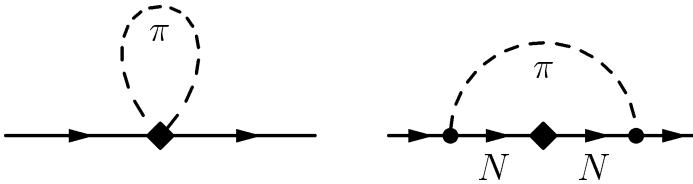
Assuming applicability of $SU(2)$ Baryon ChPT up to $m_\pi \approx 600 \text{ MeV}$:



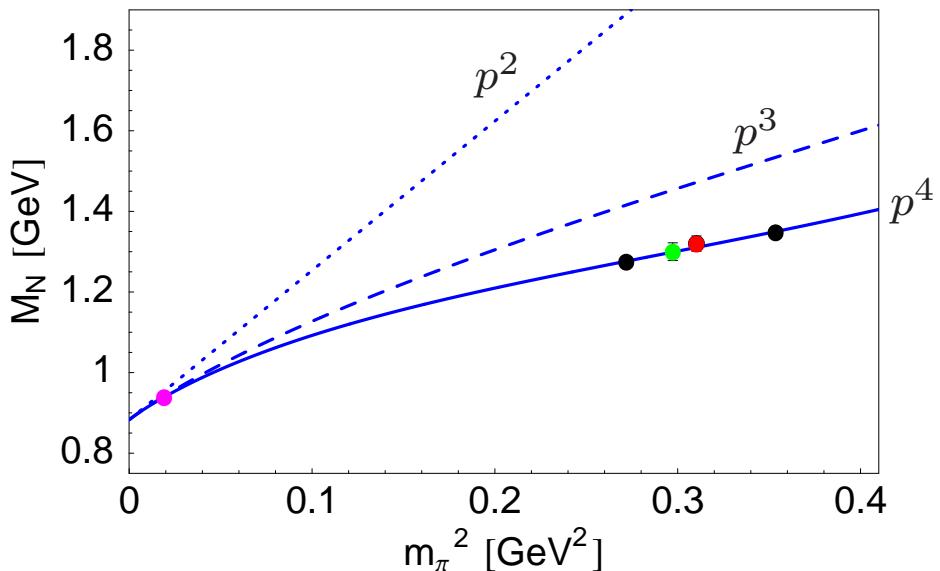
FIT	4 pts + phys.
$M_0 \text{ [GeV]}$	0.891(4)
$c_1 \text{ [GeV}^{-1}]$	-0.79(5)
$e_1^r(\lambda = 1 \text{ GeV}) \text{ [GeV}^{-3}]$	0.9(1)

M_N to next-to-leading one-loop $\mathcal{O}(p^4)$

πN



$$\begin{aligned}
 M_N = & M_0 - 4c_1 m_\pi^2 - \frac{3g_A^{0\ 2}}{32\pi f_\pi^{0\ 2}} m_\pi^3 + \left[4e_1^r(\lambda) + \frac{3c_2}{128\pi^2 f_\pi^{0\ 2}} - \frac{3g_A^{0\ 2}}{64\pi^2 f_\pi^{0\ 2} M_0} \right. \\
 & \left. - \frac{3}{32\pi^2 f_\pi^{0\ 2}} \left(\frac{g_A^{0\ 2}}{M_0} - 8c_1 + c_2 + 4c_3 \right) \ln \frac{m_\pi}{\lambda} \right] m_\pi^4 + \frac{3g_A^{0\ 2}}{256\pi f_\pi^{0\ 2} M_0^2} m_\pi^5 + \mathcal{O}(m_\pi^6)
 \end{aligned}$$



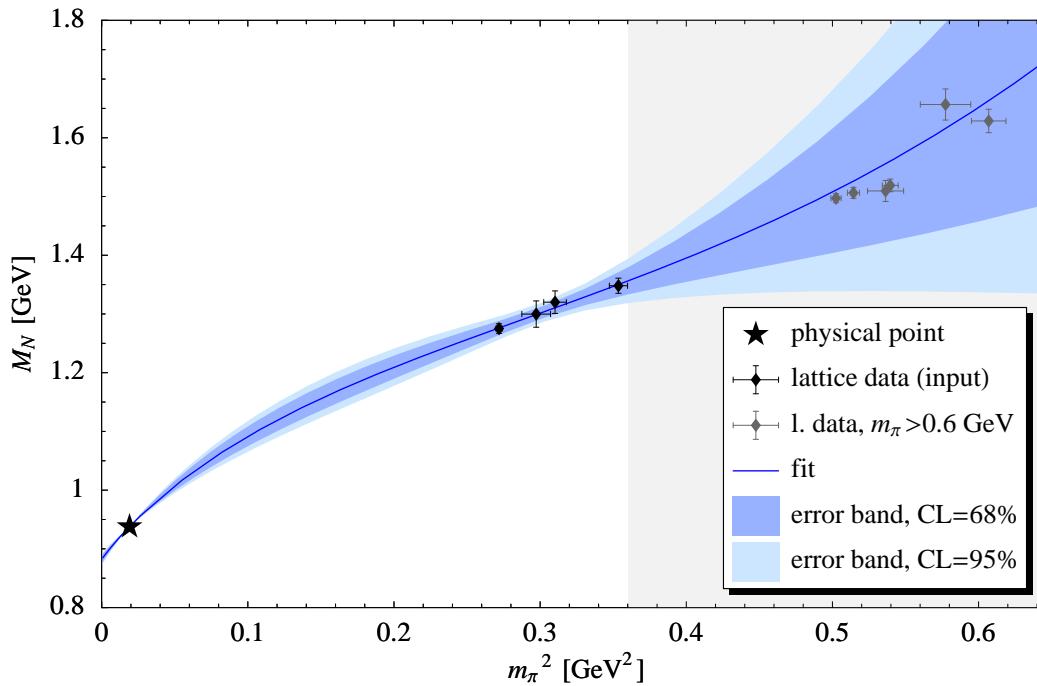
FIT	4 pts + phys.
M_0 [GeV]	0.883(3)
c_1 [GeV $^{-1}$]	-0.93(4)
$e_1^r(\lambda = 1 \text{ GeV})$ [GeV $^{-3}$]	0.7(2)

Agreement with FITS in a finite volume
(QCDSF-UKQCD 2004)

$\rightarrow \sigma_N = 49(3) \text{ MeV}$ vs $\sigma_N^{\text{emp}} = 45(8) \text{ MeV}$ (GLS 91)

M_N to $\mathcal{O}(p^4)$: statistical analysis

Error bands associated with 68% and 95% joint confidence regions for fit parameters



To extract information in the region of small quark masses

- include phenomenological input or
- perform **simultaneous fits** of several observables with a common subset of LECs

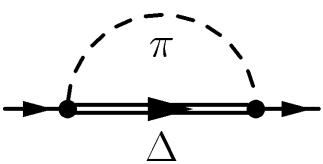
M_N to leading one-loop $\mathcal{O}(\epsilon^3)$ in SSE

$\pi N \Delta$

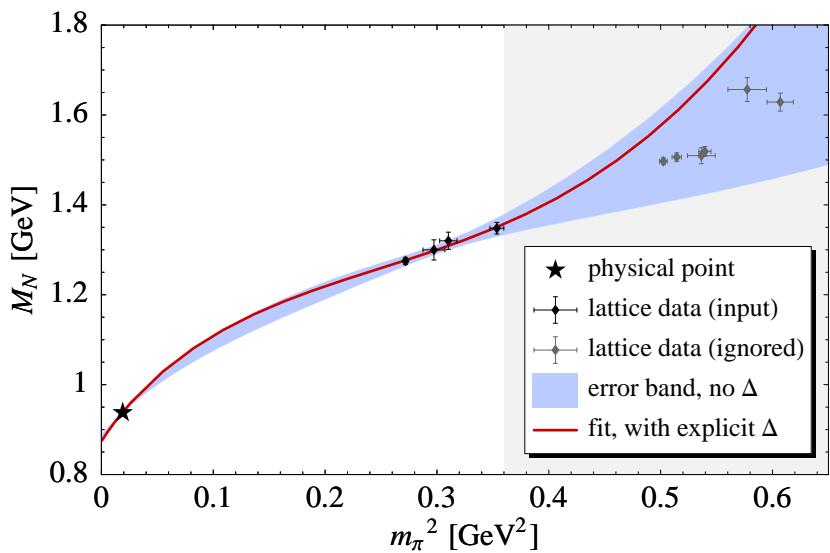
$SU(2)$ Baryon ChPT IR with spin-3/2 fields

(BHM 2003)

$$\epsilon \in \{p, m_\pi, \Delta = M_\Delta^0 - M_0\}$$



and $-4 \hat{e}_1 m_\pi^4 \bar{\Psi} \Psi$



FIT	4 pts + phys.
M_0 [GeV]	0.873(4)
c_1 [GeV $^{-1}$]	-1.08(5)
$\hat{e}_1^r(\lambda = 1 \text{ GeV})$ [GeV $^{-3}$]	2.8(6)

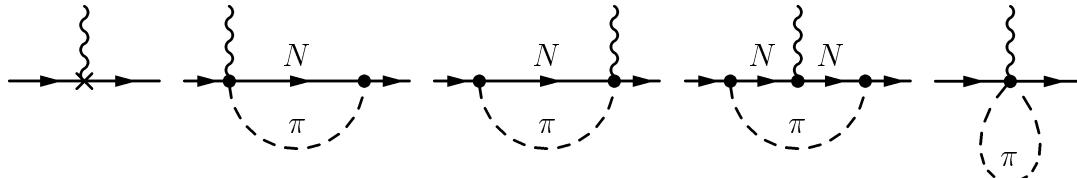
↪ Explicit treatment of Δ (1232) is not essential for $M_N(m_\pi)$

g_A to one-loop in $SU(2)$ BChPT

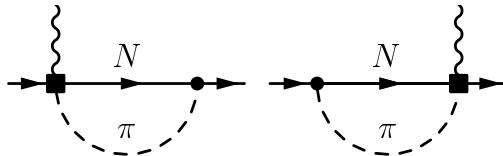
πN

$$g_A = \lim_{Q^2 \rightarrow 0} G_A(Q^2) \quad \text{and} \quad g_A^{\text{emp}} = 1.267 \pm 0.003$$

$\mathcal{O}(p^3)$



$\mathcal{O}(p^4)$



- $N_f = 2$ lattice QCD data RBCK collaboration (2003)

$a \approx 0.15 \text{ fm}$, $m_\pi L \geq 4.8$, $m_\pi \leq 650 \text{ MeV}$

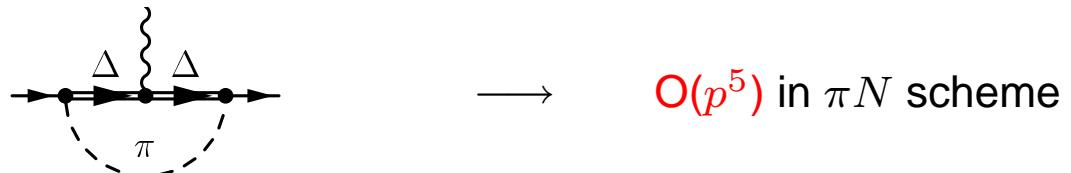
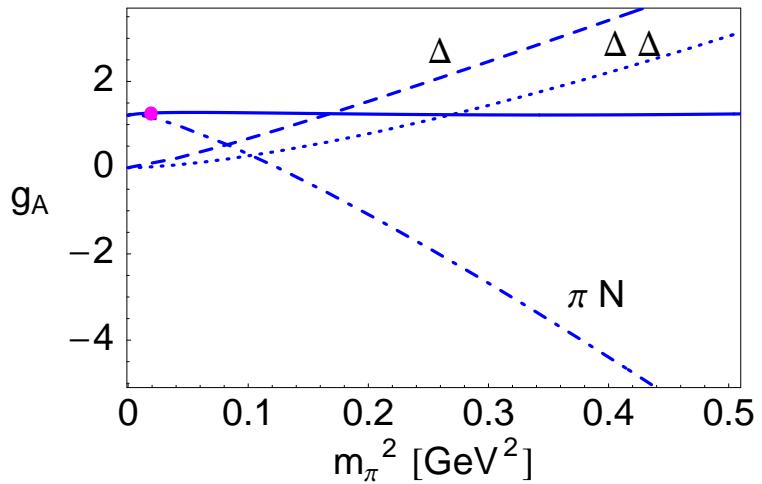
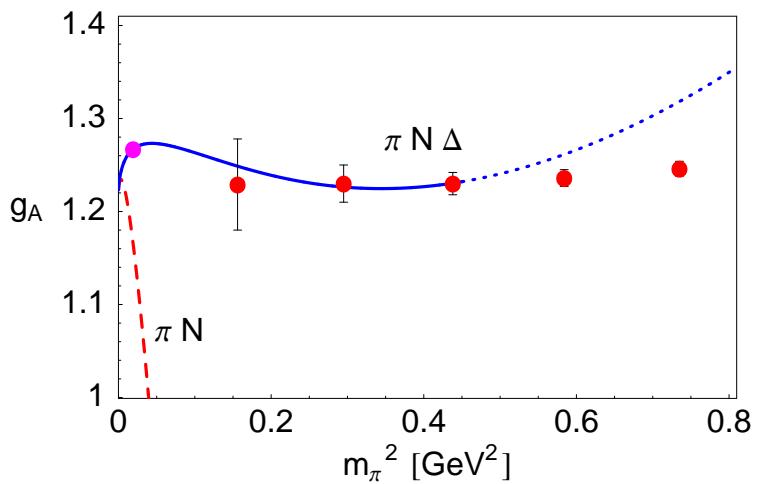
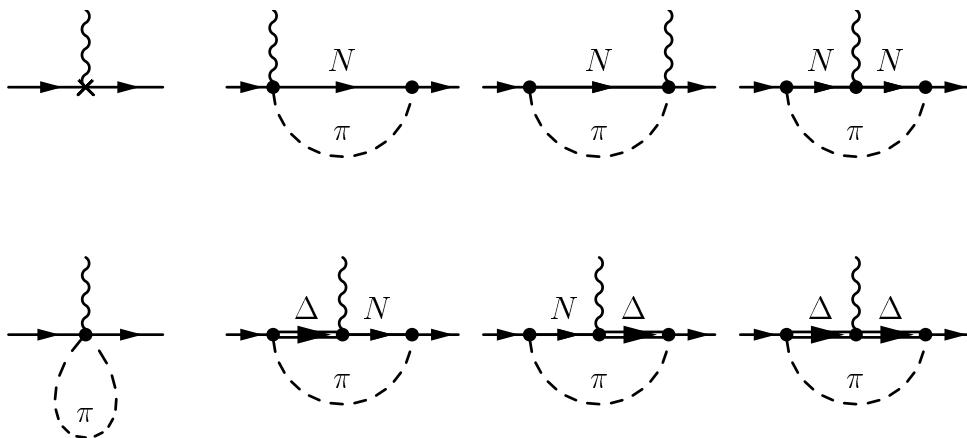
- Both at $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$: no successful interpolation possible

... but intermediate Δ (1232) contributions are important for axial properties of baryons ...

g_A and Adler-Weisberger sum rule

g_A to leading one-loop $\mathcal{O}(\epsilon^3)$ in SSE

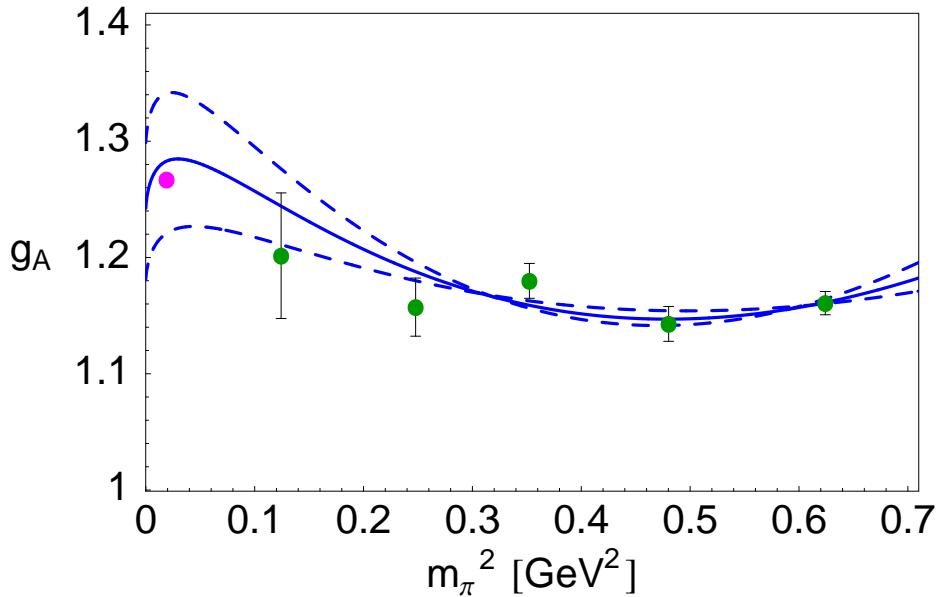
$\pi N \Delta$



g_A to leading one-loop $\mathcal{O}(\epsilon^3)$ in SSE

$\pi N \Delta$

Preliminary LHPC – MILC data (D. Richards, priv. comm. & hep-lat/0510062)



Fit parameters:

- g_A^0
- axial- Δ - Δ coupling

Linear combination of B_9 , B_{20} constrained from $\pi N \rightarrow \pi\pi N$ (FBM 2000)

Summary

- Applying Baryon Chiral Perturbation Theory to a quark mass region accessible to present full-QCD lattice simulations we obtain sensible interpolation functions at the one-loop level both for M_N and g_A .
- Inclusion of explicit Δ (1232) not decisive for M_N but crucial for g_A .
- Parameters not fixed by chiral symmetry show high degree of consistency with phenomenology.
- For M_N we see convergent hierarchy of higher-order contributions up to pion masses around 600 MeV.
- Incorporating information from $\pi N \rightarrow \pi\pi N$ dynamics, a successful chiral extrapolation for g_A has been obtained at leading one-loop order in the Small Scale Expansion.

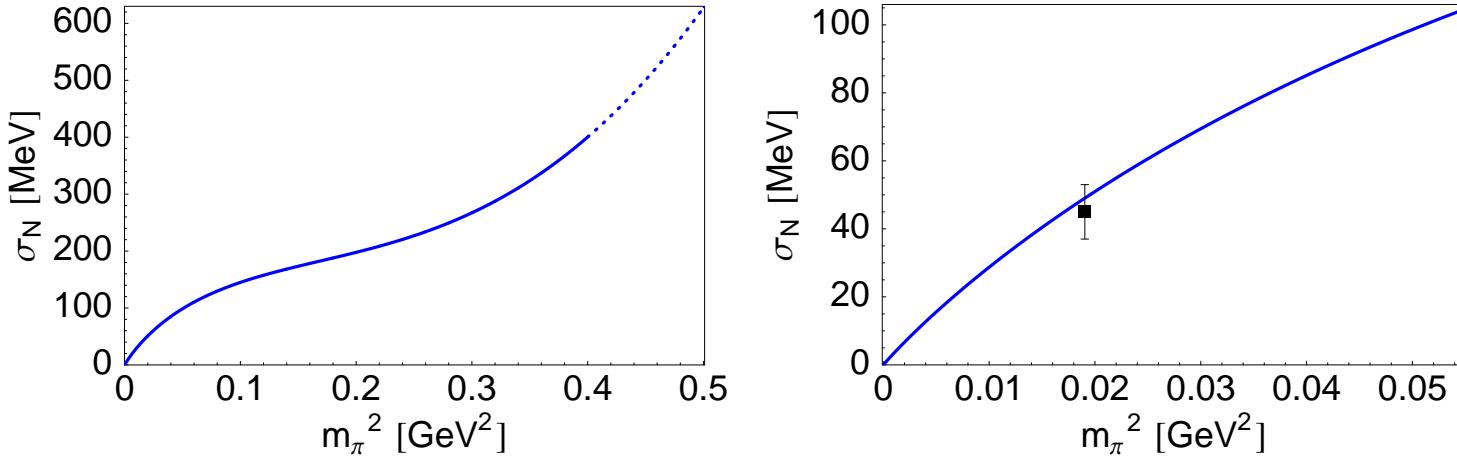
Pion-nucleon sigma term σ_N

$$\sigma_N = \langle N(\vec{p}) | m_u \bar{u}u + m_d \bar{d}d | N(\vec{p}) \rangle = \sum_{q=u,d} m_q \frac{\partial M_N}{\partial m_q} \simeq m_\pi^2 \frac{\partial M_N}{\partial m_\pi^2}$$

GOR relation:

$$m_\pi^2 = \frac{|\langle 0 | \bar{u}u | 0 \rangle_0|}{f_\pi^{0^2}} (m_u + m_d)$$

Using best fit results for $M_N(m_\pi)$ at order p^4 :

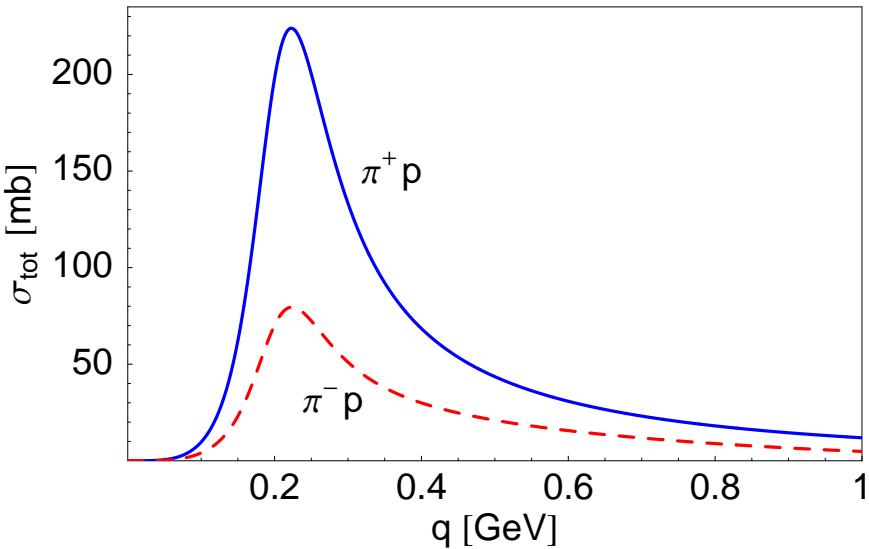


↪ $\sigma_N = 49 \pm 3 \text{ MeV}$ vs $\sigma_N^{\text{emp}} = 45 \pm 8 \text{ MeV}$ (GLS 91)

g_A and Adler-Weisberger sum rule

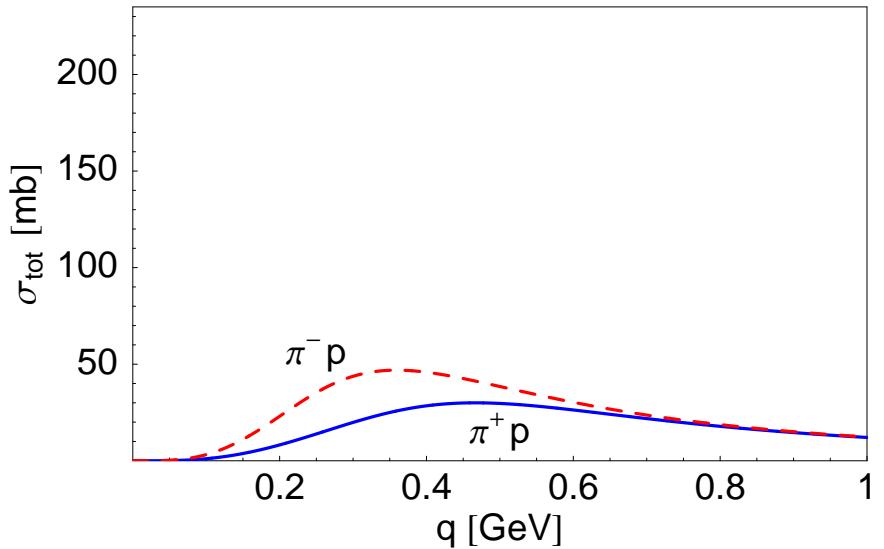
$$g_A^2 = 1 + \frac{2f_\pi^2}{\pi} \int_0^\infty \frac{dq}{\omega} [\sigma_{\pi^+ p}(\omega) - \sigma_{\pi^- p}(\omega)] + \mathcal{O}\left(\frac{m_\pi^2}{M_N^2}\right)$$

Direct and crossed nucleon and
 Δ (1232) pole graphs



$$\hookrightarrow g_A = 1.24$$

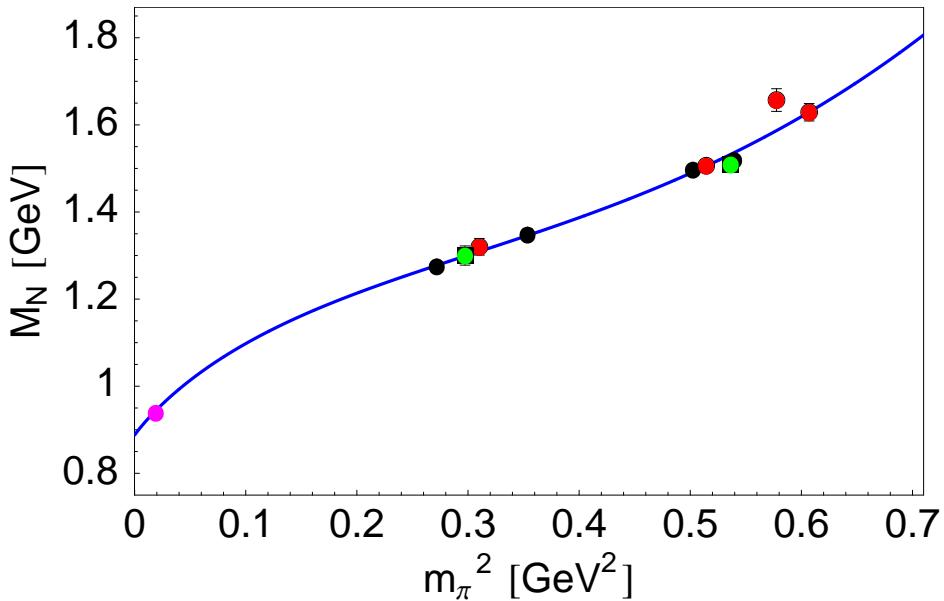
Switching off the $\pi N \Delta$ coupling



$$\hookrightarrow g_A = 0.94$$

M_N chiral extrapolation at $\mathcal{O}(p^4)$

$$\begin{aligned}
 M_N = & M_0 - 4 \textcolor{red}{c}_1 m_\pi^2 - \frac{3g_A^{0^2}}{32\pi f_\pi^{0^2}} m_\pi^3 + \left[4 e_1^r(\lambda) + \frac{3\textcolor{blue}{c}_2}{128\pi^2 f_\pi^{0^2}} - \frac{3g_A^{0^2}}{64\pi^2 f_\pi^{0^2} M_0} \right. \\
 & \left. - \frac{3}{32\pi^2 f_\pi^{0^2}} \left(\frac{g_A^{0^2}}{M_0} - 8\textcolor{red}{c}_1 + \textcolor{blue}{c}_2 + 4\textcolor{blue}{c}_3 \right) \ln \frac{m_\pi}{\lambda} \right] m_\pi^4 + \frac{3g_A^{0^2}}{256\pi f_\pi^{0^2} M_0^2} m_\pi^5 + \mathcal{O}(m_\pi^6)
 \end{aligned}$$



$$M_N(m_\pi^{\text{phys}}) = 933 \pm 62 \text{ MeV}$$